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FINANCE NOTE

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# Hedging decisions with price and output uncertainty

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**Abstract** We generally show that the introduction of output uncertainty does not affect the hedging position of the firm.

**Keywords** Hedging · Output uncertainty · Price uncertainty

**JEL Classification Numbers** D80 · D82

## 1 Introduction

The literature that studied the firm's behavior with hedging and production decisions focused on price uncertainty. Far fewer studies dealt with hedging under multiple sources of uncertainty (including output uncertainty). Examples include Viaene and Zilcha (1998), Li and Vukina (1998), Lapan and Moschini (1994), Honda (1983), Grant (1985), and Newbwry and Stiglitz (1981). However, none of these studies generally showed the impact of the output risk on the hedging position of the firm. Particularly, they did not provide the conditions under which the firm over-hedges, under-hedges, or full-hedges. In the absence of the output risk, Holthasen (1979) provided these conditions. He linked the hedging positions to the structure of the forward price [the firm over-hedges, under-hedges, or full-hedges if the forward price is upward bias (contango), downward bias (normal backwardation), or unbiased, respectively]. But similar conditions are lacking in the presence of output uncertainty. Losq (1982), under the assumptions of a fixed expected output and statistical independence between price and output, showed that the sign of the ratio of expected output to the non-speculative component of the hedging (not the total hedging) is dependent on the functional form of the utility.

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In this note, without assuming a fixed expected output, specific utility function and statistical independence, we show the impact of the output risk on the hedging position. We also link the hedging position to the structure of the forward price. We show that the introduction of the output risk will not change the hedging position of the firm. That is, in the presence of output uncertainty, the firm also over-hedges, under-hedges, or full-hedges if the forward price is upward bias, downward bias, or unbiased, respectively.

## 2 The model and results

The firm maximizes the expected utility of the profit. The profit function is specified by

$$\pi = pQ(\mathbf{x}, \eta) + (b - p)h - \mathbf{wx}$$

$$p = \bar{p} + \sigma\varepsilon;\ E\varepsilon = 0;\ \bar{Q} = Q(\mathbf{x}, E\eta)$$

where $E$ is the expectation operator, $Q(\mathbf{x}, \eta)$ is the random output, $\bar{Q}$ is the deterministic output, $\eta$ is random, $\mathbf{x}$ is the input vector, $\mathbf{w}$ is the input price vector, $h$ is the hedging, $p$ is the stochastic spot price with mean $\bar{p}$ and variance $\sigma^2$, $b$ is the non-random forward price.

It is established in the literature that the separation property does not hold under output uncertainty (see, for example, Grant 1985; Viaene and Zilcha 1998). Under price uncertainty only, the separation property was verified by Holthasen (1979), among others.

The superscripts $^\circ$ and $^*$ denote the optimal values in the absence and presence of output uncertainty, respectively.

**Proposition.** $\bar{Q}(\mathbf{x}^*) - h^* = \bar{Q}(\mathbf{x}^\circ) - h^\circ \gtreqless 0$ *if* $\bar{p} - b \gtreqless 0$.

*Proof* Without the output risk, the profit function is $\hat{\pi} = py + (b - p)h - c(y)$, where $y$ is output and $c$ is the cost function. Define $A \equiv Eu'\left(\hat{\pi}\right)\left(p - c'(y)\right)$ and $B \equiv Eu'\left(\hat{\pi}\right)(b - p)$, where $u$ is the utility function; thus (holding the parameters constant) $dA = H_{yy}\,dy + H_{yh}\,dh$ and $dB = H_{hh}\,dh + H_{yh}\,dy$ in response to the introduction of the output risk (that is, when $A$ changes from $A(y^\circ)$ to $A(y^*)$ and $B$ changes from $B(y^\circ)$ to $B(y^*)$), where $H_{hh} = Eu''\left(\hat{\pi}\right)(b - p)^2 = -H_{yh} = H_{yy} + c''(y)\,Eu' < 0$ (see Holthasen 1979).Using a matrix form, we obtain

$$\begin{bmatrix} H_{yy} & H_{yh} \\ H_{yh} & H_{hh} \end{bmatrix} \begin{bmatrix} dy \\ dh \end{bmatrix} = \begin{bmatrix} dA \\ dB \end{bmatrix}$$

Solving for $dy$ and $dh$, we obtain

$$dy = \frac{H_{hh}(dA + dB)}{H}, \tag{1}$$

$$dh = \frac{H_{hh}\,(dA + dB) - c''\,(y)\,Eu'dB}{H} = dy - \frac{c''\,(y)\,Eu'dB}{H}, \qquad (2)$$

where $H$ is the determinant of the Hessian with output certainty. Recognizing that $dB = H_{hh}\,(dh - dy)$ and substituting this in (2) , we obtain

$$dh\left(1 + \frac{c''\,(y)\,Eu'}{H}\right) = dy\left(1 + \frac{c''\,(y)\,Eu'}{H}\right),$$

thus $dy = dh$ in response to the introduction of output risk. Hence, $d\,(y - h) = 0$; that is, $y - h$ does not change in response to the output risk ($y^\circ - h^\circ = y^* - h^* = \bar{Q}\,(\mathbf{x}^*) - h$). In the absence of the output risk, $y^\circ - h^\circ \gtreqless 0$ if $\bar{p} - b \gtreqless 0$ (Holthausen, 1979) and hence $\bar{Q}\,(\mathbf{x}^*) - h^* \gtreqless 0$ if $\bar{p} - b \gtreqless 0$ in the presence of the output risk. □

It is worth noting that Grant (1985) argued that the impact of the output risk on the optimal hedging position and output is generally indeterminate. Assuming normality and constant relative risk aversion, Lapan and Moschini (1994) showed that the introduction of the output risk reduces the non-speculative component of $h$. Losq (1982) showed the same result under the assumptions of the statistical independence (between price and output) and $u''' > 0$. In addition, both Losq (1982) and Lapan and Moschini (1994) assumed that expected output is a fixed parameter. Viaene and Zilcha (1998), using an open-economy and a double-hedging model, showed that the introduction of the output risk reduces the optimal output if the marginal product is non-decreasing in the output risk and if price and output are statistically independent.

## 3 Conclusion

We showed that the introduction of the output risk does not affect the initial hedging position of the firm. That is, in the presence of the output risk, the firm continues to over-hedge, under-hedge, or full-hedge if the forward price is upward bias, downward bias, or unbiased, respectively.

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